

## Multiphase dynamics of oscillating-water-column ocean wave energy converters

Suhith Illesinghe<sup>1</sup>, Md. Kamrul Hasan<sup>1</sup> and Richard Manasseh<sup>1,2</sup>

<sup>1</sup>Faculty of Engineering and Industrial Sciences  
Swinburne University of Technology, VIC 3122, Melbourne, Australia

<sup>2</sup>Department of Mechanical Engineering  
University of Melbourne, VIC 3010, Melbourne, Australia

### Abstract

Oscillating Water Column (OWC) machines were the first-established class of Wave Energy Converters (WECs) for extracting renewable energy from ocean waves. However, as with most other classes of WECs, the underlying fluid dynamics of OWCs is poorly understood, impeding model assessments of comparative efficiency and thus demanding expensive full-scale sea trials. Most WECs are intended to have a natural frequency that, ideally, resonates at incoming wave frequencies. Therefore, modelling seeks to reduce their complex fluid dynamics to linearisable ordinary differential equation models, enabling engineering design.

A multiphase fluid dynamics model of an OWC is presented that includes the compressibility of internal gases, and a preliminary parameterisation of the Reynolds stresses and interfacial dissipation arising from the peculiar nature of reciprocating turbulence. On reduction to integral form, it was found that two linear eigenmodes arise in all OWC systems: a “bubble acoustic” mode in which gas compressibility dominates, and a “pendulum” mode in which gravity dominates.

### Introduction

An Oscillating Water Column (OWC) in its simplest form is a vertical tube that is fixed in an inertial reference frame, with one end immersed under a water surface, and the other end open to the atmosphere. It behaves dynamically as a liquid pendulum. If the water in the tube is displaced upwards and released, it will fall back, overshoot its equilibrium position and oscillate until dissipation damps out the motion. Its frequency of oscillation is  $\sqrt{g/L}$  rad s<sup>-1</sup>, where  $L$  is the length of tube that is underwater, and  $g$  is the acceleration due to gravity [1].

Systems based on the OWC principle have been utilised to generate electricity since the 1980s; however, it is mainly in the last 15 years that a number of OWC prototypes have been developed and deployed worldwide [2]. Like most other Wave Energy Converter (WEC) concepts, OWCs are intended to have a natural frequency of oscillation that is “tuned” to be close to the frequency of the ocean waves. When the device resonates, maximal energy is extracted from the ocean. Most OWCs use a bi-directional turbine mounted in the air above the water column to extract energy from the motion [2].

A key issue with all resonating WECs, including the OWC, is the dissipation in the system. As with any damped oscillator, if the system has low damping, the resonant peak would be very high, delivering very high power; but this would drop to low power when the ocean wave frequency shifts from the resonant ideal. In contrast, a system with high damping would deliver much less power at resonance, but would not cause a large drop in output if the wave frequency were to drift off-resonance. Wave frequencies vary with geography, season, and

distant weather. Knowledge of the damping of WECs is critical to predicting their economic performance in any given location.

Prior approaches to OWC dynamics have modelled the internal dynamics of the water column with integral forms of the simplified Navier-Stokes equations or Euler equations. Typical examples include the works of [3]. This has generally been done implicitly, by directly writing down mass-spring-damping equations with the parameters unknown and to be determined, presumably, by future experiment. Thus, the PDEs of momentum and mass conservation have been modelled as ODEs which are useful for the analysis of oscillator dynamics, and thus to the semi-analytic estimation of efficiencies of practical interest.

Derivation of the approximate natural frequency of an OWC from first principles is straightforward, and as a first approximation,  $\sqrt{(g/L)/(2\pi)}$  might be sufficient for prototype design. However, derivation of the damping is not straightforward, and hitherto, the extensive literature on OWCs generally assumes arbitrary values. This has meant that quantitative prediction of the most important aspect of OWC performance - its actual power output - has not been possible.

In this paper, we outline how to model the fluid dynamics of an OWC from first principles. We note that appropriate experimental data is not available, particularly on the turbulent dissipation, but we identify factors that could be given numerical values if appropriate laboratory experiments were done in the future. We also explicitly show how the gas compressibility can provide an additional and separate natural frequency. We do not explicitly consider a turbine or other power extraction device, since the focus is on the natural internal dynamics of the OWC.

### Formulation

A peculiar feature of dissipation in an OWC is that the fluid flow within it is reciprocating. A reciprocating fluid flow is an unsteady flow that completely reverses periodically, so the mean flow is zero. The fact that the mean flow is zero is of particular significance when the turbulence responsible for much of the loss of energy from the system is considered. Reciprocating turbulence is a complex and interesting topic beyond the scope of the present paper, but it will have to be treated somehow. The experimental results of [4] use a reciprocating version of the normal engineering friction factor. This approach and data could be adapted to calculate the damping of OWC. However, in order to utilise this or any other treatment of reciprocating turbulence, careful thought must first be given to the normally straightforward procedure of deriving the Reynolds stresses. To avoid confusion with the term ‘mean flow’, we will use the term *primary flow* to refer to fluid flows inside the device oscillating with periods typical of ocean waves.

In previous work on OWCs, the equations of motion have either been dimensional, or analyses have begun with simplified ver-

sions of the integral forms of the equations of motion in which scaling decisions were implicit. With OWCs, there are a great variety of regimes of geometry and operating parameters, for which different and potentially inconsistent formulations would be necessary. As OWC designs proliferate, it is particularly important to explicitly and rigorously consider the scaling parameters, as a prelude to reducing the internal fluid dynamics to the ODEs that are necessary for effective modelling of the system.

The mass and momentum conservations in the multiphase flow within the OWC are given by,

$$\frac{\partial \rho^*}{\partial t^*} + \nabla_j^*(\rho^* u_j^*) = 0, \quad (1)$$

$$\frac{\partial(\rho^* u_i^*)}{\partial t^*} + \nabla_j^*(\rho^* u_i^* u_j^*) = -\nabla_i^* p^* + \nabla_j^* \tau_{ij}^*, \quad (2)$$

where  $\tau_{ij}^*$  is the viscous stress tensor which can be represented for an isotropic fluid as

$$\tau_{ij}^* = \mu \left( \nabla_j^* u_i^* + \nabla_i^* u_j^* - \frac{2}{3} \nabla_k^* u_k^* \delta_{ij} \right). \quad (3)$$

Here the asterisks denote dimensional quantities, and  $u_i^*$  is the velocity vector,  $p^*$  is the pressure,  $\mu$  is the dynamic viscosity, assumed constant, and  $\rho^*$  is the density. In the following, the density of the liquid will be assumed constant, precluding shocks or sound waves in the liquid, but the density of any gas within the OWC device will be allowed to vary in time only. This again precludes shocks or sound within the gas, but does permit trapped (or partially trapped) gas volumes to be compressed and expanded. Gas compressibility is a feature widely recognised in the literature to provide a ‘spring’ stiffness in an OWC [5].

In a cylindrical co-ordinate system,  $\mathbf{x}^* = (x^*, r^*, \phi)$ , the length scaling is

$$\mathbf{x}^* = \mathbf{L}\mathbf{x}, \quad (4)$$

where  $\mathbf{L}$  is the length-scaling matrix given by

$$\mathbf{L} = \begin{bmatrix} L & 0 & 0 \\ 0 & D & 0 \\ 0 & 0 & D \end{bmatrix},$$

in which  $L$  is the length scale of the device in the main direction in which water flows in response to the driving waves, the  $x$ -direction, and  $D$  is the length scale at right angles to this main direction. For example, for a pipe-like OWC,  $L$  may be 10s of metres, while the pipe diameter  $D$  may be a few metres. However, some OWCs in current use have  $D \simeq 10$  m. The dynamically-significant variables are scaled with the bar representing the primary flow, while primes represent fluctuating variables in the usual way,

$$\begin{aligned} t^* &= \omega^{-1} t \\ \mathbf{u}^* &= \hat{\xi}_F \omega (\bar{\mathbf{U}} + I\mathbf{u}'), \\ p^* &= \frac{\rho_f g L D}{\hat{\xi}_F} (\bar{P} + I p'), \\ \rho^* &= \rho_f \rho \end{aligned} \quad (5)$$

in which  $\omega$  is the angular frequency of the ocean wave,  $\hat{\xi}_F$  is the amplitude scale of the motion, for example, the amplitude of the ocean waves which may be a few metres,  $I$  is a ‘turbulence intensity’ and  $\rho_f$  is the liquid density. The turbulence intensity in non-reciprocating pipe flows is usually an empirical function of the Reynolds number, and is used in numerical calculations [6]; while an analogous definition may be applicable to a reciprocating system, for now,  $I$  is simply a small parameter.

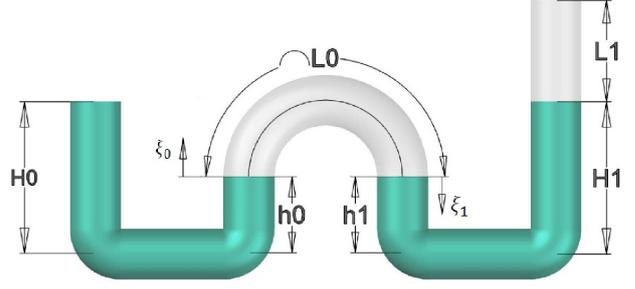


Figure 1: Different zones in an OWC in which the aspect ratio  $L/D$  is large, making it ‘pipe-like’. The  $x$  direction follows the central axis of the curved pipe-line; gas-liquid interfaces are meant to be horizontal. In this configuration, there are two liquid zones (green regions) of length  $S_0$  and  $S_1$  which trap a gas bubble or void of length  $L_0$  between them.

For the multiphase aspect of the system, the Volume of Fluid (VOF) Eulerian-Eulerian concept is employed (though numerical calculations are not the intent of the present paper). In the VOF approach, the overall non-dimensional density is given by,

$$\rho = C + \rho_g(1 - C), \quad (6)$$

where  $C$  is the liquid volume fraction and  $\rho_g = \frac{\rho_g^*}{\rho_f}$ , is the non-dimensional density of the gas (most likely air, though other gases could be used in voids trapped in OWCs). The volume fraction is given by

$$C(\mathbf{x}) = \begin{cases} 1 & \text{if } \mathbf{x} \text{ is in liquid} \\ 0 & \text{if } \mathbf{x} \text{ is in gas} \end{cases},$$

As noted earlier, the density of the liquid,  $\rho_f$ , is assumed constant, while the density of the gas,  $\rho_g$ , is allowed to vary with time only. The overall multiphase density  $\rho$  can vary with time, not only because  $\rho_g$  can vary, but also because the interface location can vary in an inertial reference frame. Moreover, the interface location could vary not only owing to interface motion with the primary frequency  $\omega$ , but owing to turbulent motions. In this regard, lateral (at right angles to  $x$ ) ‘sloshing’ waves inside the OWC could also occur on much smaller scales than the primary rise-and-fall due to the ocean waves. Numerical calculations of multiphase flow would normally include such motions, but since the present paper seeks an analytic simplification to ODEs in  $x$  only, it is necessary to parameterise such motions. These interfacial fluctuations were treated by [7] by assuming the phase-indicator function  $C$  is also composed of primary and fluctuating components, i.e.,

$$C = \bar{C} + c' \quad (7)$$

#### Ensemble averaging

Applying the ensemble averaging, as usual, terms in the variables with primes with powers greater than one will disappear [8]. Applying the scalings of (5) and performing the ensemble averaging to (1)-(3) gives the averaged mass conservation equation as

$$\frac{\partial \bar{\rho}}{\partial t} + \frac{\hat{\xi}_F}{D} \nabla_j \left[ \bar{\rho} \bar{U}_j + I(1 - \rho_g) \overline{u_j' c'} \right] = 0, \quad (8)$$

and the momentum conservation equation as

$$\begin{aligned} & \frac{\partial}{\partial t} [\bar{U}_i \bar{\rho} + I(1 - \rho_g) \overline{(u'_i c')}] + \frac{\hat{\xi}_F}{D} \nabla_j [\bar{\rho} \bar{U}_i \bar{U}_j + I^2 \bar{\rho} \overline{(u'_i u'_j)}] \\ & + I(1 - \rho_g) \bar{U}_j \overline{(u'_i c')} + I(1 - \rho_g) \bar{U}_i \overline{(u'_j c')} \\ & = -\frac{1}{\hat{F}r} \nabla_i \bar{P} + \frac{1}{\text{Re}_\omega} \nabla_j \left( \nabla_j \bar{U}_i + \nabla_i \bar{U}_j - \frac{2}{3} \nabla_k \bar{U}_k \delta_{ij} \right), \end{aligned} \quad (9)$$

where  $\bar{\rho} = \bar{C} + \rho_g(1 - \bar{C})$  and  $\nabla = \frac{D}{L} \frac{\partial}{\partial x} \bar{e}_x + \frac{\partial}{\partial r} \bar{e}_r + \frac{1}{r} \frac{\partial}{\partial \phi} \bar{e}_\phi$ .

Two dimensionless numbers have been introduced in the above non-dimensional momentum equation: a Froude number,  $\hat{F}r = (\omega^2 \hat{\xi}_F^2) / (gL)$  and the kinetic Reynolds number,  $\text{Re}_\omega = (\rho_0 \omega D^2) / \mu$ .

### Separation of the OWC into zones

We choose to model the OWC in four separate zones: external liquid (the open ocean water), internal liquid only, internal gas only, and an internal interface zone which can contain both liquid and gas. The zones are matched by the pressure at their boundaries. The external liquid zone contributes inertia but not significant dissipation to the system. In the internal liquid zone, assumed incompressible as noted earlier, viscous and turbulent dissipation cause significant damping of the overall OWC dynamics. In contrast, in the gas zone, momentum (and thus loss of momentum to dissipation) is negligible, but the gas compressibility must be taken into account. Thus, the role of the gas zone is effectively to alter the pressure boundary condition on the interface zone and thus the liquid zone. Finally, in the interface zone, few approximations are justifiable, apart from the simplification of the interface to zero thickness, in which case its dynamics disappear. The interface zone may be needed in a comprehensive OWC model if sloshing motions (waves inside the OWC) cause significant dissipation. It is mentioned for completeness, although it will be neglected in the present paper.

In the internal liquid zone,  $\bar{C} = 1$  and  $c' = 0$ , giving  $\bar{\rho} = 1$ . As a consequence, the conservation equations in the liquid zone can be reduced to

$$\begin{aligned} & \nabla_i \bar{U}_i = 0, \quad (10) \\ & \frac{\partial \bar{U}_i}{\partial t} + \frac{\hat{\xi}_F}{D} (\bar{U}_j \nabla_j \bar{U}_i) = -\frac{1}{\hat{F}r} \nabla_i \bar{P} \\ & + \nabla_j \left[ \frac{1}{\text{Re}_\omega} \nabla_j \bar{U}_i - I^2 \frac{\hat{\xi}_F}{D} \overline{(u'_i u'_j)} \right]. \end{aligned} \quad (11)$$

Consider only the dynamically-significant  $x$ -component of equation (11),

$$\begin{aligned} & \frac{\partial \bar{U}}{\partial t} + \frac{\hat{\xi}_F}{D} \left[ \frac{D}{L} \bar{U} \frac{\partial \bar{U}}{\partial x} + \bar{V} \frac{\partial \bar{U}}{\partial r} + \frac{\bar{W}}{r} \frac{\partial \bar{U}}{\partial \phi} \right] = -\frac{1}{\hat{F}r} \frac{D}{L} \frac{\partial \bar{P}}{\partial x} \\ & + \frac{1}{\text{Re}_\omega} \left[ \frac{D^2}{L^2} \frac{\partial^2 \bar{U}}{\partial x^2} + \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial \bar{U}}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 \bar{U}}{\partial \phi^2} \right] \\ & - I^2 \frac{\hat{\xi}_F}{D} \left[ \frac{D}{L} \frac{\partial \overline{(u'^2)}}{\partial x} + \frac{1}{r} \frac{\partial}{\partial r} \left( r \overline{(u'v')} \right) + \frac{1}{r} \frac{\partial \overline{(u'w')}}{\partial \phi} \right]. \end{aligned} \quad (12)$$

Now, assume the ratio  $L/D$  is large, so the internal liquid zone of the OWC is 'pipe-like'. As noted earlier, some currently operating OWCs have a large  $D$ , similar to  $L$ , for which this assumption would not be valid. In addition, fully developed flows

have been assumed throughout the liquid zones, neglecting regions of developing flow, bends and curvature effects. Further assuming an axisymmetric pipe flow, i.e. derivatives with respect to  $\phi$  are zero, the momentum equation in the  $x$ -direction now becomes

$$\begin{aligned} \frac{\partial \bar{U}}{\partial t} = & -\frac{1}{\hat{F}r} \frac{D}{L} \frac{\partial \bar{P}}{\partial x} \\ & - I^2 \frac{\hat{\xi}_F}{D} \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \overline{(u'v')} \right) \right] + \frac{1}{\text{Re}_\omega} \left[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial \bar{U}}{\partial r} \right) \right]. \end{aligned} \quad (13)$$

The last two terms of (13) are the Reynolds and viscous shear stress, respectively; these are now combined into a total shear stress, giving

$$\frac{\partial \bar{U}}{\partial t} = -\frac{1}{\hat{F}r} \frac{D}{L} \frac{\partial \bar{P}}{\partial x} - \frac{1}{r} \frac{\partial (r \tau_r)}{\partial r}, \quad (14)$$

where the total shear stress is given by

$$\tau_r = I^2 \frac{\hat{\xi}_F}{D} \overline{(u'v')} - \frac{1}{\text{Re}_\omega} \frac{\partial \bar{U}}{\partial r}. \quad (15)$$

For generality, several liquid and gas zones are permitted. There must always be a first vertical internal-liquid column between the device mouth and the first gas zone. There may also be a second internal-liquid column, with the gas trapped at the top of the two liquid columns in a header; and the second column could be a U-tube, permitting a second gas zone, and so on. Any of the gas zones (in the simplest OWC configuration, there is only one liquid and one gas zone) could drive the turbine (the power-take-off device), though for practicalities of maintenance it would probably be the last gas (air) zone exhausting directly to atmosphere that drives the power-take off. As noted earlier, for clarity we concentrate on the elementary natural frequencies of the system, and thus do not treat the power-take-off device. Similarly, insignificant mass of external ocean water is assumed to be set into motion outside the device mouth; but a potential-flow calculation to estimate this added mass would be feasible.

### Integration and reduction to ODE form

On integration in  $r$ , variables  $\xi_0$  and  $\xi_1$  can be defined. These are, respectively, the displacement of the radial-mean boundaries between the first mass of internal liquid and the first gas zone, and the displacement of the boundary between the first gas zone and a second internal liquid zone as shown in figure 1. In this example, the interface is assumed to be of zero thickness. Integrating (14) times  $2\pi r$  with respect to  $r$  from 0 to 0.5 and then dividing by  $\pi(0.5)^2$  gives,

$$\frac{d\bar{U}_b}{dt} = -\frac{1}{\hat{F}r} \frac{D}{L} \frac{d\bar{P}}{dx} - 4\tau_r. \quad (16)$$

The paucity of relevant experimental data on reciprocating turbulence is now the issue. Following the experimentally-validated approach of [9], the velocity gradient in reciprocating pipe flow is simply modelled as

$$\beta \frac{\bar{U}_b}{((D/2)/D)} = 2\beta \bar{U}_b, \quad (17)$$

where  $\beta$  is a factor which depends on  $\text{Re}_\omega$ . The above-mentioned total stress can now be defined as  $\tau_r = \mu_t (2\beta \bar{U}_b)$ , where  $\mu_t$  is a 'total viscosity' (dimensionless) that includes both molecular viscosity and an eddy viscosity for reciprocating turbulence, an example of which was given by [9]. In reality, the data of [9] is unlikely to be directly applicable to a full-scale OWC. Thus,  $\mu_t$  is a key parameter which will demand

specialised experimental and numerical investigations in the future for the geometry of the OWC. This approach gives rise to a linear damping term. A quadratic damping term could also be employed as noted in [5], and although quadratic damping would be consistent with the ‘friction factor’ approach of [4], the experiment of [4] is even less relevant to the OWC than that of [9].

The boundary conditions on the liquid zones require the pressure in the gas. This is given by the ideal-gas law, which relates pressure to volume and hence to the displacements  $\xi_0$  and  $\xi_1$ . A final assumption is that the compression and expansion of the gas is small, permitting linearisation of the ideal-gas law. (No linearisation of the liquid-zone equation was necessary since the geometric assumptions that lead to (13) eliminated the advective term in the primary velocity.) Upon linearisation of the ideal-gas expression, and finally integration in  $x$ , the internal liquid-zone equations reduce to

$$\begin{aligned} \ddot{\xi}_0 + B_0 \dot{\xi}_0 + A_{00} \xi_0 - A_{01} \xi_1 &= \frac{D}{S_0} \frac{1}{\hat{F}_r} F(t), \\ \ddot{\xi}_1 + B_1 \dot{\xi}_1 + A_{11} \xi_1 - A_{10} \xi_0 &= 0, \end{aligned} \quad (18)$$

where  $F(t)$  represents the force produced by ocean waves at the mouth, and

$$\begin{aligned} A_{00} &= \frac{D}{S_0} \frac{1}{\hat{F}_r} \frac{\gamma \bar{P}_0 \hat{\xi}_F}{L_0} + \frac{g}{\omega^2 S_0}, & A_{01} &= \frac{D}{S_0} \frac{1}{\hat{F}_r} \frac{\gamma \bar{P}_0 \hat{\xi}_F}{L_0}, \\ A_{11} &= \frac{D}{S_1} \frac{1}{\hat{F}_r} \frac{\gamma \bar{P}_0 \hat{\xi}_F}{L_0} + \frac{2g}{\omega^2 S_1}, & A_{10} &= \frac{D}{S_1} \frac{1}{\hat{F}_r} \frac{\gamma \bar{P}_0 \hat{\xi}_F}{L_0}, \\ B_0 &= 8\mu_t \beta_0, & B_1 &= 8\mu_t \beta_1, \end{aligned}$$

where  $\beta_0$  and  $\beta_1$  are the values of  $\beta$  for the liquid zones,

$$\bar{P}_0 = \bar{P}_{atm} + \hat{F}_r \frac{H_0 - h_0}{D} \frac{g}{\hat{\xi}_F \omega^2} = \bar{P}_{atm} + \hat{F}_r \frac{H_1 - h_1}{D} \frac{g}{\hat{\xi}_F \omega^2},$$

and  $S_0$ ,  $L_0$ ,  $S_1$ ,  $H_0$ ,  $H_1$ ,  $h_0$  and  $h_1$  are dimensional lengths defined in figure 1, and  $\gamma$  is the ratio of specific heats for the gas, assumed to be adiabatically compressed.

## Results

Figure 2 shows an example of the response of a ‘pipe-like’ OWC device with one trapped ‘bubble’ or void and two liquid columns on either side, as drawn in figure 1. It can be seen in that there are two roots corresponding to the eigenvalues of (18). The range of frequencies chosen corresponds to typical ocean swell with periods of the order of 10 s. The ‘pendulum’ mode frequency is about 0.13 Hz. However, gas compressibility, enforced in this example by trapping gas between two water masses, introduces a second degree of freedom, creating a second mode of oscillation. The existence of a second mode of oscillation corresponds to a ‘bubble-acoustic’ mode in which the gas is compressed and expanded.

## Conclusion

Oscillating water columns are a simple fluid resonators that can extract power from ocean waves. Ideally, predicting their performance requires ODE models of their dynamics, which hitherto have been heuristic in form without clarity on the exact assumptions employed. We have determined the assumptions needed to rigorously derive an ODE model of the fluid elements of an OWC. Dissipative terms due to reciprocating turbulence have been identified. Crude approximations to these can be extracted from existing literature. Above all, however, future experimental programmes on reciprocating multiphase turbulence are warranted and can be guided by the needs of the present model.

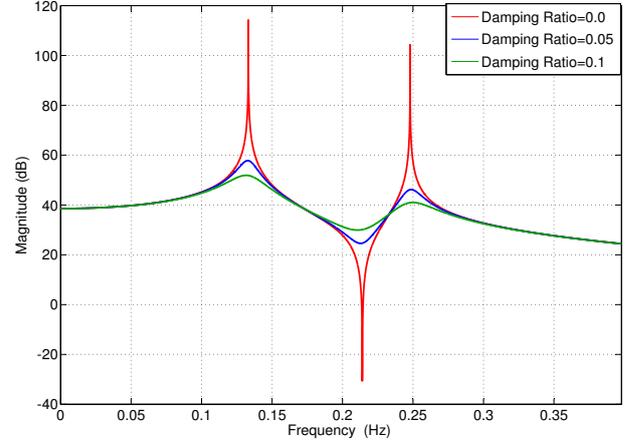


Figure 2: Frequency response plot (Bode magnitude diagram) for parameters  $S_0 = S_1 = 20$  m,  $D = 2$  m and for the machine mounted close to the surface, so the ambient pressure and temperature are standard atmospheric. The zero-damping curve (in red) has been truncated at its peaks.

It is shown that the model admits two modes, one due to the ‘pendulum’ like behaviour of the OWC, and one due to the ‘bubble-acoustic’ like behaviour of the OWC.

Neighbouring OWCs may be coupled by sea-surface wave radiation, leading to multiple eigenmodes. This understanding may lead to methods for optimising OWCs and multiply-connected ‘farms’ of OWCs to maximise efficiency.

## References

- [1] Streeter, V. L. and Wylie, E. B. (1979). *Fluid Mechanics*. McGraw-Hill International.
- [2] Falcão, A. F. O. (2010). Wave energy utilization: A review of the technologies. *Renewable and Sustainable Energy Reviews*, 14:899–918.
- [3] Evans, V. D. (1982). Wave-power absorption by systems of oscillating surface pressure distributions. *Journal of Fluid Mechanics*, 114:481–499.
- [4] Zhao, T. S. and Cheng, P. (1996). Experimental studies on the onset of turbulence and frictional losses in an oscillatory turbulent pipe flow. *International Journal of Heat and Fluid Flow*, 17:356–362.
- [5] Olvera, A. Prado, E. and Czitrom, S. (2007). Parametric resonance in an oscillating water column. *Journal of Engineering Math*, 57:1–21.
- [6] Pope, S. B. *Turbulent Flows*. Cambridge University Press.
- [7] Shirani, E. Ghadiri, F. and Ahmadi, A. (2011). Modeling and Simulation of Interfacial Turbulent Flows. *Journal of Applied Fluid Mechanics*, 4:43–49.
- [8] Silva, U. D. Zhang, J. Orou, J. C. and Johnson, J. A. (1996). Evidence of natural closure in compressible turbulent flow. *Journal of Physics D: Applied physics*, 29:263–266.
- [9] Ogawa, A. and others (2007). Damping Vibrational Motion of Liquid Column in Vertical U-Tube for Newtonian and non-Newtonian Liquids. *8th International Symposium on Experimental and Computational Aerothermodynamics of Internal Flows Lyon, ISAI8-009*